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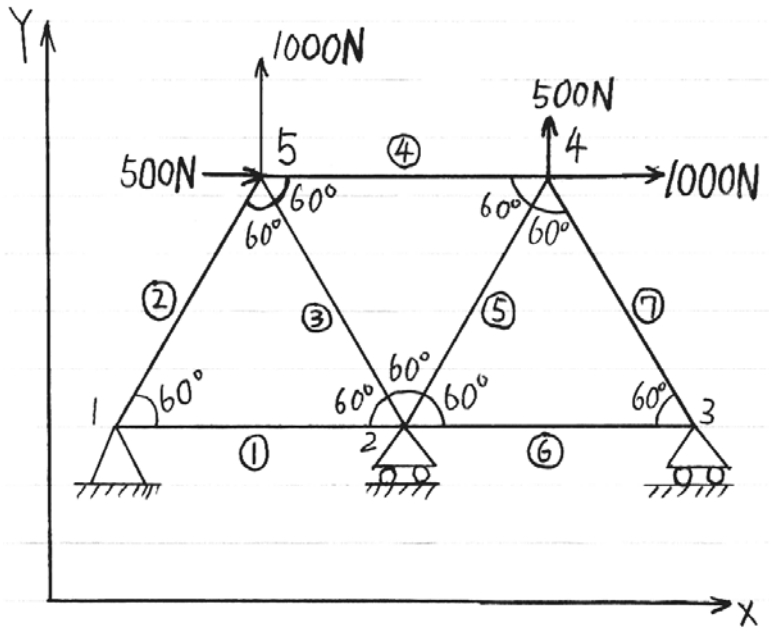

図 1

図 1 に示すトラス構造の全体剛性マトリックスを求める。

部材①の節点 1 を i、節点 2 を j とすると θ =0 ° で

$$K_{11}^{\ 1} = K_{22}^{\ 1} = k\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad K_{12}^{\ 1} = K_{21}^{\ 1} = k\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

部材②では i=1、j=5 とすると θ =60 ° で

$$K_{11}^{\ 2} = K_{55}^{\ 2} = k\begin{bmatrix} \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix} \qquad K_{15}^{\ 2} = K_{51}^{\ 2} = k\begin{bmatrix} -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{3}{4} \end{bmatrix}$$

部材③では i＝2、j＝5 とすると θ ＝120 ° で

$$K_{22}^{\ 3} = K_{55}^{\ 3} = k\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix} \qquad K_{25}^{\ 3} = K_{52}^{\ 3} = k\begin{bmatrix} -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{3}{4} \end{bmatrix}$$

部材④では i=5、j=4 とすると θ＝0°で

$$K_{55}^{\ 4} = K_{44}^{\ 4} = k\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad K_{54}^{\ 4} = K_{45}^{\ 4} = k\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

部材⑤では i=2、j=4 とすると θ＝60°で

$$K_{22}^{\ 5} = K_{44}^{\ 5} = k\begin{bmatrix} \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix} \qquad K_{24}^{\ 5} = K_{42}^{\ 5} = k\begin{bmatrix} -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{3}{4} \end{bmatrix}$$

部材⑥では i=2、j=3 とすると θ＝0°で

$$K_{22}^{\ 6} = K_{33}^{\ 6} = k\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad K_{23}^{\ 6} = K_{32}^{\ 6} = k\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

部材⑦では i=3、j=4 とすると θ＝120°で

$$K_{33}^{\ 7} = K_{44}^{\ 7} = k\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix} \qquad K_{34}^{\ 7} = K_{43}^{\ 7} = k\begin{bmatrix} -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{3}{4} \end{bmatrix}$$

となる。全体剛性マトリックスの大きさは 10×10 となり、これらを加えると

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \\ F_{x5} \\ F_{y5} \end{Bmatrix} = k \begin{bmatrix} \frac{5}{4} & \frac{\sqrt{3}}{4} & -1 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} \\ -1 & 0 & \frac{5}{2} & 0 & -1 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} \\ 0 & 0 & -1 & 0 & \frac{5}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{3}{2} & 0 & -1 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & \frac{3}{2} & 0 & 0 \\ -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 & -1 & 0 & \frac{3}{2} & 0 \\ -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \\ U_4 \\ V_4 \\ U_5 \\ V_5 \end{Bmatrix}$$

ここで$U_1 = 0$、$V_1 = 0$、$V_2 = 0$、$V_3 = 0$なので

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \\ 500 \\ 500 \\ 1000 \end{Bmatrix} = k \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{2} & 0 & -1 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & \frac{5}{4} & 0 & -\frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & \frac{3}{2} & 0 & -1 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{4} & 0 & \frac{\sqrt{3}}{4} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & -1 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{\sqrt{3}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \\ U_4 \\ V_4 \\ U_5 \\ V_5 \end{Bmatrix}$$

このマトリックスを計算して

$$\frac{5}{2}U_2 - U_3 - \frac{1}{4}U_4 - \frac{\sqrt{3}}{4}V_4 - \frac{1}{4}U_5 + \frac{\sqrt{3}}{4}V_5 = 0$$

$$-U_2 + \frac{5}{4}U_3 - \frac{1}{4}U_4 + \frac{\sqrt{3}}{4}V_4 = 0$$

$$-\frac{1}{4}U_2 - \frac{1}{4}U_3 + \frac{3}{2}U_4 - U_5 = \frac{1000}{k}$$

$$-\frac{\sqrt{3}}{4}U_2 + \frac{\sqrt{3}}{4}U_3 + \frac{3}{2}V_4 = \frac{500}{k}$$

$$-\frac{1}{4}U_2 - U_4 + \frac{3}{2}U_5 = \frac{500}{k}$$

$$\frac{\sqrt{3}}{4}U_2 + \frac{3}{2}V_5 = \frac{1000}{k}$$

これらを連立させると

$$U_2 = \frac{35625 - 5125\sqrt{3}}{33k} \qquad U_3 = \frac{15500 - 2500\sqrt{3}}{11k} \qquad U_4 = \frac{15375 - 875\sqrt{3}}{6k}$$

$$U_5 = \frac{146625 - 8125\sqrt{3}}{66k} \qquad V_4 = \frac{24375 - 3625\sqrt{3}}{66k} \qquad V_5 = \frac{49125 - 11875\sqrt{3}}{66k}$$

となる。よって

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \\ F_{x5} \\ F_{y5} \end{Bmatrix} = k \begin{bmatrix} \frac{5}{4} & \frac{\sqrt{3}}{4} & -1 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} \\ -1 & 0 & \frac{5}{2} & 0 & -1 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} \\ 0 & 0 & -1 & 0 & \frac{5}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{3}{2} & 0 & -1 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & \frac{3}{2} & 0 & 0 \\ -\frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 & -1 & 0 & \frac{3}{2} & 0 \\ -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \frac{35625 - 5125\sqrt{3}}{33k} \\ 0 \\ \frac{15500 - 2500\sqrt{3}}{11k} \\ 0 \\ \frac{15375 - 875\sqrt{3}}{6k} \\ \frac{24375 - 3625\sqrt{3}}{66k} \\ \frac{146625 - 8125\sqrt{3}}{66k} \\ \frac{49125 - 11875\sqrt{3}}{66k} \end{Bmatrix}$$

となる。

次に反力を求める。

$$F_{x1} = -k \times \frac{35625 - 5125\sqrt{3}}{33k} - \frac{k}{4} \times \frac{146625 - 8125\sqrt{3}}{66k} - \frac{\sqrt{3}}{4} k \times \frac{49125 - 11875\sqrt{3}}{66k} = -1500[N]$$

$$F_{y1} = -\frac{\sqrt{3}}{4} k \times \frac{146625 - 8125\sqrt{3}}{66k} - \frac{3}{4} k \times \frac{49125 - 11875\sqrt{3}}{66k} = -1194[N]$$

$$F_{y2} = -\frac{\sqrt{3}}{4} k \times \frac{15375 - 875\sqrt{3}}{6k} - \frac{3}{4} k \times \frac{24375 - 3625\sqrt{3}}{66k} + \frac{\sqrt{3}}{4} k \times \frac{146625 - 8125\sqrt{3}}{66k} - \frac{3}{4} k \times \frac{49125 - 11875\sqrt{3}}{66k}$$

$$= -661[N]$$

$$F_{y3} = -\frac{\sqrt{3}}{4} k \times \frac{15500 - 2500\sqrt{3}}{11k} + \frac{\sqrt{3}}{4} k \times \frac{15375 - 875\sqrt{3}}{6k} - \frac{3}{4} k \times \frac{24375 - 3625\sqrt{3}}{66k} = 355[N]$$

となる。

次に応力を求める。

部材①

$\theta = 0$

$$U_1 = 0, V_1 = 0, U_2 = \frac{35625 - 5125\sqrt{3}}{33k}, V_2 = 0$$

$$\delta = \left\{ \frac{35625 - 5125\sqrt{3}}{33k} - 0 \right\} \times 1 = \frac{L\left(35625 - 5125\sqrt{3}\right)}{33EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{35625 - 5125\sqrt{3}}{33EA}$$

$$\sigma^1 = E\varepsilon = \frac{35625 - 5125\sqrt{3}}{33A}$$

部材②

$\theta = 60$

$$U_1 = 0, V_1 = 0, U_5 = \frac{146625 - 8125\sqrt{3}}{66k}, V_5 = \frac{49125 - 11875\sqrt{3}}{66k}$$

$$\delta = \left\{ \frac{146625 - 8125\sqrt{3}}{66k} - 0 \right\} \times \frac{1}{2} + \left\{ \frac{49125 - 11875\sqrt{3}}{66k} - 0 \right\} \times \frac{\sqrt{3}}{2} = \frac{L\left(111000 + 41000\sqrt{3}\right)}{132EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{111000 - 41000\sqrt{3}}{132EA}$$

$$\sigma^2 = E\varepsilon = \frac{111000 + 41000\sqrt{3}}{132A}$$

部材③

$\theta = 120$

$$U_2 = \frac{35625 - 5125\sqrt{3}}{33k}, V_2 = 0, U_5 = \frac{146625 - 8125\sqrt{3}}{66k}, V_5 = \frac{49125 - 11875\sqrt{3}}{66k}$$

$$\delta = \left\{ \frac{146625 - 8125\sqrt{3}}{66k} - \frac{35625 - 5125\sqrt{3}}{33k} \right\} \times \left(-\frac{1}{2} \right) + \frac{49125 - 11875\sqrt{3}}{66k} \times \frac{\sqrt{3}}{2} = \frac{L\left(-87250 + 67500\sqrt{3} \right)}{132EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-87250 + 67500\sqrt{3}}{132EA}$$

$$\sigma^3 = E\varepsilon = \frac{-87250 + 67500\sqrt{3}}{132A}$$

部材④

$\theta = 0$

$$U_5 = \frac{146625 - 8125\sqrt{3}}{66k}, V_5 = \frac{49125 - 11875\sqrt{3}}{66k}, U_4 = \frac{15375 - 875\sqrt{3}}{6k}, V_4 = \frac{24375 - 3625\sqrt{3}}{66k}$$

$$\delta = \left\{ \frac{15375 - 875\sqrt{3}}{6k} - \frac{146625 - 8125\sqrt{3}}{66k} \right\} \times 1 = \frac{L\left(22500 - 17750\sqrt{3} \right)}{66EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{22500 - 17750\sqrt{3}}{66EA}$$

$$\sigma^4 = E\varepsilon = \frac{22500 - 17750\sqrt{3}}{66A}$$

部材⑤

$\theta = 60$

$$U_2 = \frac{35625 - 5125\sqrt{3}}{33k}, V_2 = 0, U_4 = \frac{15375 - 875\sqrt{3}}{6k}, V_4 = \frac{24375 - 3625\sqrt{3}}{66k}$$

$$\delta = \left\{ \frac{15375 - 875\sqrt{3}}{6k} - \frac{35625 - 5125\sqrt{3}}{33k} \right\} \times \frac{1}{2} + \frac{24375 - 3625\sqrt{3}}{66k} \times \frac{\sqrt{3}}{2} = \frac{L\left(87000 + 25000\sqrt{3} \right)}{132EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{87000 + 25000\sqrt{3}}{132EA}$$

$$\sigma^5 = E\varepsilon = \frac{87000 + 25000\sqrt{3}}{132A}$$

部材⑥

$\theta = 0$

$$U_2 = \frac{35625 - 5125\sqrt{3}}{33k}, V_2 = 0, U_3 = \frac{15500 - 2500\sqrt{3}}{11k}, V_3 = 0$$

$$\delta = \left\{ \frac{35625 - 5125\sqrt{3}}{33k} - \frac{15500 - 2500\sqrt{3}}{11k} \right\} \times 1 = \frac{L\left(-10875 + 2375\sqrt{3}\right)}{33EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-10875 + 2375\sqrt{3}}{33EA}$$

$$\sigma^6 = E\varepsilon = \frac{-10875 + 2375\sqrt{3}}{33A}$$

部材⑦

$\theta = 120$

$$U_3 = \frac{15500 - 2500\sqrt{3}}{11k}, V_3 = 0, U_4 = \frac{15375 - 875\sqrt{3}}{6k}, V_4 = \frac{24375 - 3625\sqrt{3}}{66k}$$

$$\delta = \left\{ \frac{15375 - 875\sqrt{3}}{6k} - \frac{15500 - 2500\sqrt{3}}{11k} \right\} \times -\frac{1}{2} + \frac{24375 - 3625\sqrt{3}}{66k} \times \frac{\sqrt{3}}{2} = \frac{L\left(65250 + 29750\sqrt{3}\right)}{132EA}$$

$$\varepsilon = \frac{\delta}{L} = \frac{65250 + 29750\sqrt{3}}{132EA}$$

$$\sigma^7 = E\varepsilon = \frac{65250 + 29750\sqrt{3}}{132A}$$

となる。

以上より

$$U_2 = \frac{35625 - 5125\sqrt{3}}{33k} \qquad U_3 = \frac{15500 - 2500\sqrt{3}}{11k} \qquad U_4 = \frac{15375 - 875\sqrt{3}}{6k}$$

$$U_5 = \frac{146625 - 8125\sqrt{3}}{66k} \qquad V_4 = \frac{24375 - 3625\sqrt{3}}{66k} \qquad V_5 = \frac{49125 - 11875\sqrt{3}}{66k}$$

$$F_{x1} = -1500[N] \qquad F_{y1} = -1194[N]$$

$$F_{y2} = -661[N] \qquad F_{y3} = 355[N]$$

$$\sigma^1 = \frac{35625 - 5125\sqrt{3}}{33A} \qquad \sigma^2 = \frac{111000 + 41000\sqrt{3}}{132A} \qquad \sigma^3 = \frac{-87250 + 67500\sqrt{3}}{132k}$$

$$\sigma^4 = \frac{22500 - 17750\sqrt{3}}{66A} \qquad \sigma^5 = \frac{87000 + 25000\sqrt{3}}{132A} \qquad \sigma^6 = \frac{-10875 + 2375\sqrt{3}}{33A}$$

$$\sigma^7 = \frac{65250 + 29750\sqrt{3}}{132A}$$